

The Sznajd Consensus Model with Continuous Opinions

Santo Fortunato

Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany

e-mail: fortunat@physik.uni-bielefeld.de

Abstract

In the consensus model of Sznajd, opinions are integers and a randomly chosen pair of neighbouring agents with the same opinion forces all their neighbours to share that opinion. We propose a simple extension of the model to continuous opinions, based on the criterion of bounded confidence which is at the basis of other popular consensus models. Here the opinion s is a real number between 0 and 1, and a parameter ϵ is introduced such that two agents are compatible if their opinions differ from each other by less than ϵ . If two neighbouring agents are compatible, they take the mean s_m of their opinions and try to impose this value to their neighbours. We find that if all neighbours take the average opinion s_m the system reaches complete consensus for any value of the confidence bound ϵ . We propose as well a weaker prescription for the dynamics and discuss the corresponding results.

Keywords: Sociophysics, Monte Carlo simulations.

1 Introduction

The consensus model of Sznajd [1] has rapidly acquired importance in the new field of computational sociophysics [2, 3], where one tries to model society as a system of agents which interact with each other, with the ultimate aim to explain the occurrence at a global level of complex phenomena like the formation of hierarchies [4] and consensus [1, 5, 6, 7, 8].

In the original formulation of Sznajd, the agents sit on the sites of a one-dimensional chain, and the opinion variable s can take only the values ± 1 ('up' or 'down'). In this respect the Sznajd model can be considered the

"Ising model of opinion dynamics". Initially each agent takes opinion $+1$ with probability p and -1 with probability $1 - p$. The dynamics is based on the principle that if two friends share the same opinion, they may succeed in convincing their acquaintances of their opinion ("united we stand, divided we fall"). In the Sznajd algorithm, one randomly chooses a pair of neighbouring agents i and $i + 1$ and check whether their opinions s_i and s_{i+1} are the same ($++$ or $--$). If this is the case, their neighbours $i - 1$ and $i + 2$ take the opinion of i and $i + 1$ (so one finally has $s_{i-1} = s_i = s_{i+1} = s_{i+2}$). It may of course happen that $s_i \neq s_{i+1}$ ($+-$ or $-+$). In this case, each agent of the pair "imposes" its opinion to the neighbour of the other agent of the pair, so $s_{i-1} = s_{i+1}$ and $s_{i+2} = s_i$. This second rule has usually been neglected in the successive studies on the Sznajd model. In these works one used the so-called "basic" Sznajd dynamics, where the opinions of the neighbours of the chosen pair of agents change only if the two agents agree, otherwise nothing happens. In its basic version, the Sznajd dynamics leads to a configuration where all agents share the same opinion (consensus), for any value of the initial concentration p of (up) opinions. If $p < 1/2$ ($> 1/2$) all agents will have opinion -1 ($+1$) in the final configuration.

In this paper we will mainly deal with the basic Sznajd dynamics, but we will as well present interesting results corresponding to the original Sznajd prescription. Meanwhile a lot is known on this model. A great deal of refinements have been introduced, which can be grouped in two categories: variations of the social topology and modifications of the "convincing" rule. Several lattice topologies have been adopted, simple square [9], cubic [10], triangular [11], dilute [12], etc. Moreover, network topologies have also been investigated, like pseudo-fractal [13] and especially scale free networks [10, 14], which are currently very popular [15]. As far as the dynamics is concerned, one has explored what happens when i neighbouring sites ($i \geq 1$), not necessarily two, convince their neighbours, for the cases $i = 1$ (single site) [13, 16] and $i = 3$ [13, 14]. Furthermore, one has also studied the case where the possible number of opinion states is larger than two [10, 13, 17, 19, 20]. The interest in the Sznajd model is not simply academic, as with this model one was able to reproduce the distribution of the number of candidates according to the number of votes they received in Brazilian and Indian elections [10, 13].

Here we do not want to concentrate on specific applications or refinements of the model, but rather reformulate it for the case in which opinions are real numbers. There are two reasons why this formulation could be important:

- it deals with the case in which each individual has, at least initially, its own attitude/opinion, so one does not have to introduce the total number of possible opinions as a parameter;
- it allows a direct comparison of the Sznajd dynamics and its predictions with the other two consensus models with real opinions, that of Deffuant et al. [6] and that of Krause-Hegselmann (KH) [7].

We start from a society where the relationships between the people are represented by the edges of a graph, not necessarily a regular lattice. The first step of the algorithm consists in assigning to each agent a real number between 0 and 1 with uniform probability. After that, as in the prescription of Sznajd, we choose a pair of neighbouring agents (i, j) and compare their opinions s_i and s_j . This is the point where we need to introduce a new prescription. The opinions, being real, can never be equal, as required by the Sznajd rule, but we have to soften this condition. As a matter of fact, instead of equality, we can demand "closeness", i.e. that the two opinions must differ from each other by less than some real number ϵ . This immediately recalls the principle of Bounded Confidence which characterizes both the model of Deffuant et al. and that of KH. There the parameter ϵ is called confidence bound and, if $|s_i - s_j| < \epsilon$, the two agents are compatible, in the sense that their positions are close enough to allow a discussion (interaction) between them; the discussion leads to a modification of their positions. In our case, we shall keep the denomination of confidence bound for ϵ , but the concept acquires a slightly stronger meaning: we say that if $|s_i - s_j| < \epsilon$ the two agents are compatible enough to share the same opinion s_m after their interaction, where $s_m = (s_i + s_j)/2$. This is actually what happens in the Deffuant model when the so-called convergence parameter $\mu = 1/2$ [6]. If instead i and j are not compatible, both i and j maintain their opinions s_i and s_j .

Now we must define what happens to the opinions of the neighbours of the pair (i, j) . If i and j are not compatible, we do nothing, as in the basic version of Sznajd we mentioned above. If i and j are compatible, we devise two possible prescriptions, that we call "Strong Continuous (SC) Sznajd" and "Weak Continuous (WC) Sznajd" such that:

- in SC Sznajd, all neighbours take the opinion s_m of the pair, independently of their own opinions;
- in WC Sznajd, only the agents which are compatible with their neighbour in the pair (i, j) take the opinion s_m , where the compatibility

refers to the opinion of the neighbour center site i or j before it gets updated to s_m .

We shall see that these two prescriptions lead to very different results. We update the opinions of the agents in the following way: we make an ordered sweep over all agents, and, for each agent i , we select at random one of its neighbours and apply our version of the Sznajd prescription. We repeat the procedure over and over until we find that, after a sweep, the opinion of each agent did not change appreciably, where "appreciably" for us means by more than 10^{-9} . We remark that in all studies on the Sznajd model one usually performed random and not sequential updates: for this reason we made some tests with random updating, and the results are the same for SC Sznajd and essentially the same for WC Sznajd. In all simulations we adopted two kinds of graphs to describe society, a square lattice with periodic boundary conditions and a Barabási-Albert (BA) network [21]. A BA network with N vertices can be constructed with a simple dynamical procedure. First one has to specify the outdegree m of the vertices, i.e. the number of edges which originate from a vertex. One starts from m vertices which are all connected to each other and adds further $N - m$ vertices one at a time. When a new vertex is added, it selects m of the pre-existing vertices as neighbours, so that the probability to get linked to a vertex is proportional to the number of its neighbours.

Since one needs to fix the value of the confidence bound ϵ before starting the simulation, the results will in general depend on ϵ and we shall investigate this dependence. Let us start to present the results relative to SC Sznajd. In all simulations we have carried on, both on the lattice and on BA networks, we found that the system converges to a configuration where all agents have one and the same opinion (complete consensus), for any value of ϵ . This result, which matches that of the original discrete version, shows that the Sznajd dynamics is most effective to achieve a full synchronization of the agents. We remark that the result holds independently of the initial distribution of opinions, which needs not be uniform. We also found that the value of the final opinion s_f of the agents is not $1/2$, as in the models of Deffuant and KH, but it can take any value in a range centered at $1/2$. The width of the range and the probability distribution of s_f depend on ϵ . In Fig. 1 we show the probability distribution of s_f for a square lattice and four values of ϵ , obtained from 100000 runs. As one can see, the histograms are all symmetric with respect to the center opinion $1/2$, as expected, but their shape varies

with ϵ . We distinguish three characteristic profiles, flat, double peaked and single peaked for low, intermediate and high values of ϵ , respectively. In the case of a single peak, we have noticed that the width shrinks approximately as $1/\sqrt{N}$, when N increases; so the peak is probably doomed to become a δ -function centered at $1/2$ when $N \rightarrow \infty$. On the other hand, at low ϵ , we noticed that the histogram does not change appreciably when N increases. This means that there must be some ϵ_c such that if $\epsilon < \epsilon_c$ the final opinion s_f falls in a finite range of opinions, if instead $\epsilon > \epsilon_c$ $s_f = 1/2$.

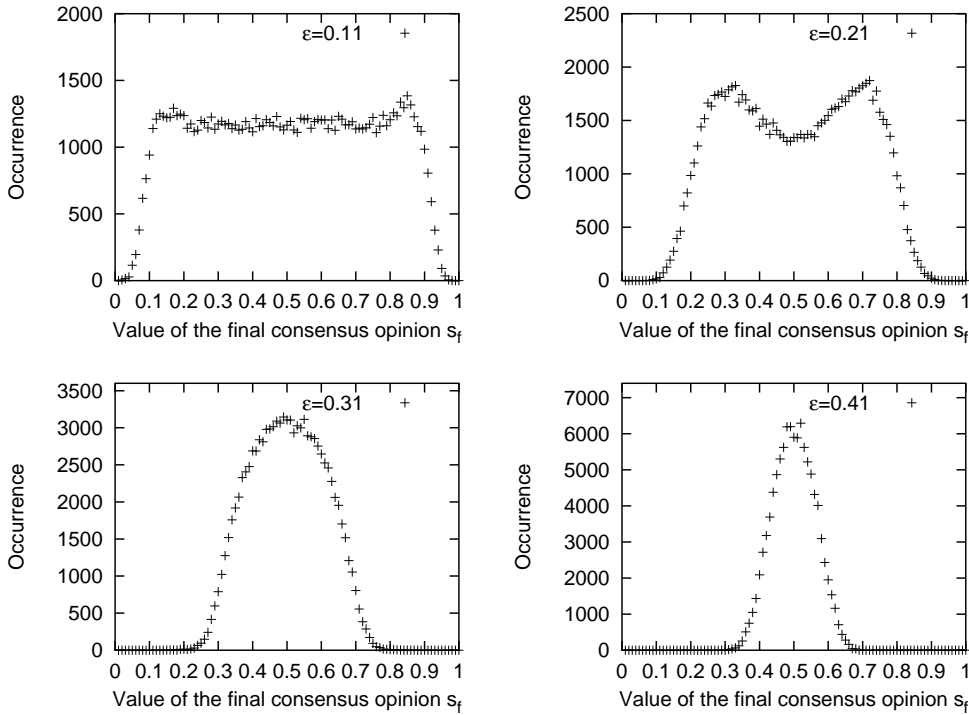


Figure 1: Probability distribution of the final surviving opinion for Strong Continuous Sznajd. The social topology is a square lattice with 2500 sites.

Since the structure of the final opinion configuration is always the same, i.e. consensus, we checked what happens if we add to the convincing rule of the basic Sznajd dynamics the "anti-ferromagnetic" prescription originally introduced in the seminal paper [1], for the case in which the opinions of the agents of the randomly selected pair (i, j) are not compatible. In this case, the extension to our case is trivial: the neighbours of i take the opinion of j

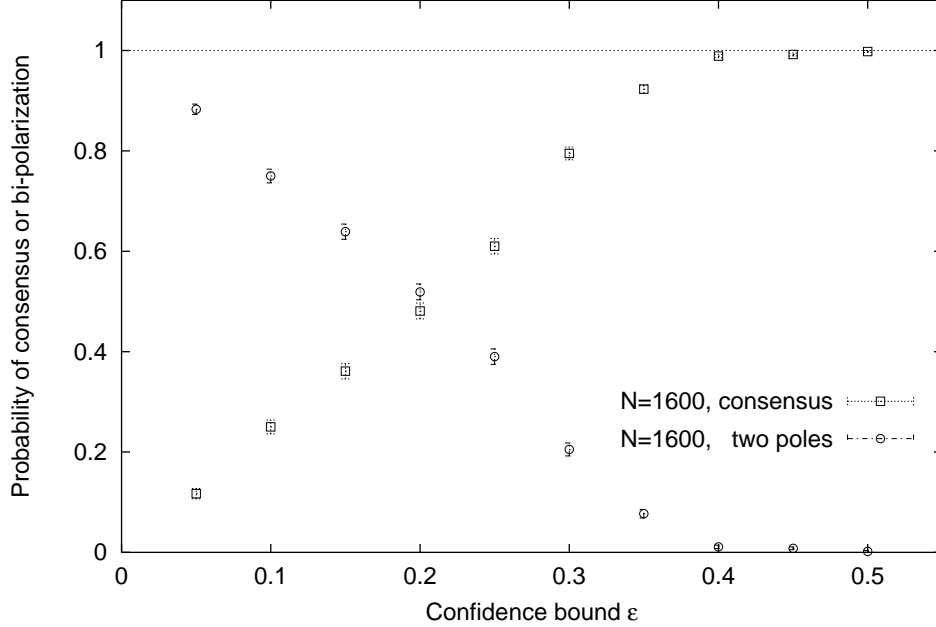


Figure 2: Fraction of samples with complete consensus and bi-polarization for Strong Continuous Sznajd with both "ferromagnetic" and "anti-ferromagnetic" coupling. The agents sit on the sites of a square lattice of side $L = 40$.

and viceversa. The effect of this more complex dynamics is that the system can converge to one of only two possible situations: either there is complete consensus, like before, or there is a perfect splitting of the community in two factions, with exactly half of the agents sharing either opinion. In this case, too, one confirms the result obtained with the discrete Sznajd model, where one would have either a perfect ferromagnet (all agents with opinions $+1$ or -1), or a perfect antiferromagnet (with the opinions $+1$ and -1 which regularly alternate in the chain/lattice). Indeed, when the population splits in two factions, the two opinions regularly alternate on the lattice, as this is the only possible stable situation different from consensus. In Fig. 2 we plot the probability of having either of the final states, i.e. the fraction of samples in which we obtained consensus or bi-polarization, for different values of ϵ . Society is a square lattice and the total number of samples is 1000. We

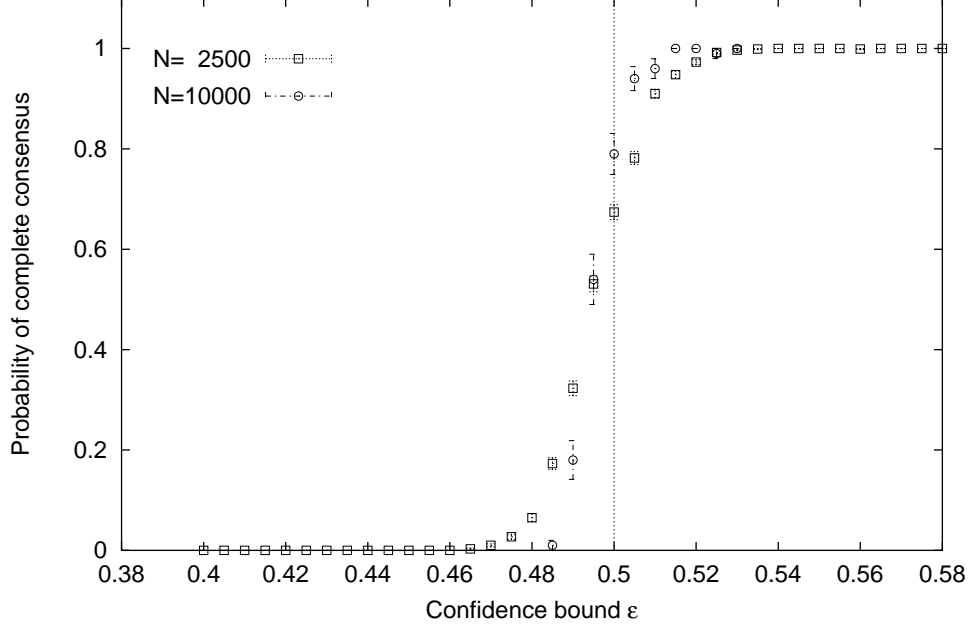


Figure 3: Fraction of samples with complete consensus as a function of ϵ , for Weak Continuous Sznajd on two square lattices with 2500 and 10000 agents.

notice that bi-polarization is very likely to occur at low values of ϵ , whereas one always obtains consensus for ϵ larger than about 0.4 (although the real threshold is probably $1/2$, as in the model of Deffuant [22]).

Now we turn to Weak Continuous Sznajd. We believe that this is a more realistic implementation of the Sznajd dynamics, as only people whose positions are somewhat close to each other can be influenced. The fact that we apply bounded confidence to the neighbours as well dramatically changes the scenario. Now one can have a variable number of opinion clusters in the final configuration, depending on the value of the confidence bound, as in the models of Deffuant and KH. First, we tried to determine the threshold for complete consensus. For this purpose we calculated again the fraction of samples with a single surviving opinion, out of 1000 total configurations, for several values of ϵ . Fig. 3 shows the results, where we took again a square lattice topology and two different sizes to investigate the limit when the number of agents goes to infinity. From the figure it is clear that the

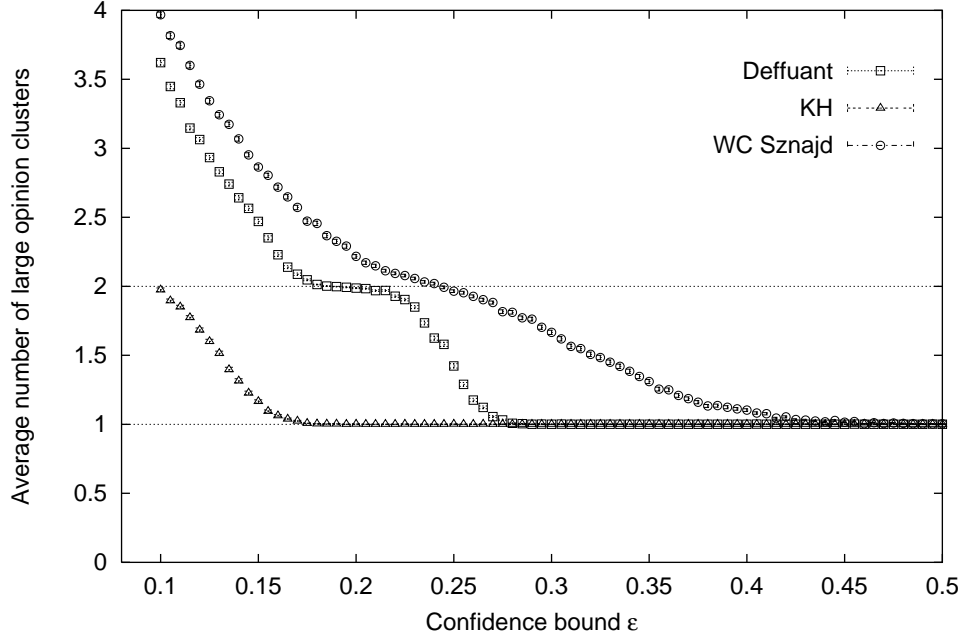


Figure 4: Average number of large opinion clusters as a function of ϵ for Weak Continuous Sznajd, Deffuant and Krause-Hegselmann. The social topology is a scale free network á la Barabási-Albert with 1000 vertices.

threshold for complete consensus is $1/2$, as in the model of Deffuant [22]. A similar analysis on Barabási-Albert networks confirms the result.

Next, we compared the model with the other two continuous opinion models, Deffuant and KH. One of the most important issues is the variation with ϵ of the number of clusters in the final configuration. We decided to focus on large clusters: we say that a cluster is large if it includes more than \sqrt{N} agents, where N is as usual the size of the total population. As a matter of fact, especially when N is not too large, as in the cases we have examined, it quite often happens that in the final configuration several clusters with very few agents co-exist with larger ones. Most small clusters are artefacts due to the finite size of the system, and would disappear if N becomes large. That is why we focus on large clusters, which represent most of the real parties/factions created by the dynamics in the limit $N \rightarrow \infty$. Fig. 4 shows the pattern of the large cluster multiplicity with ϵ , for WC Sznajd, Deffuant

and KH, respectively. The system is a scale free network á la Barabási-Albert, with 1000 vertices. Further simulations at larger N indicate that the pattern shown in the figure is nearly asymptotic, i.e. does not change appreciably when N increases. We see that there is a sort of monotonic relationship between the three models: for a given ϵ there are more large clusters in the final configuration for WC Sznajd than for Deffuant, and more for Deffuant than for KH. In particular one has to go to much higher values of ϵ for WC Sznajd in order to obtain a single large cluster in the final configuration, a situation which is instead much easier to reach for the other two models.

In conclusion, we have presented a generalization of the Sznajd dynamics to real-valued opinions, based on bounded confidence. We proposed two prescriptions for updating the opinions, which differ from each other by the influence of the randomly selected pair of (compatible) agents on their neighbours. According to the first rule, all neighbours accept the average opinion of the pair. In this case, the fate of the system is simple: all agents will end up with the same opinion at some stage. The second rule, instead, limits the influence of the pair only to those neighbours which are compatible with their friend in the pair. In this case one can have any number of opinion clusters in the final configuration, depending on ϵ , and consensus is attained only for $\epsilon > 1/2$. The latter prescription turns out to be less effective to create large opinion clusters than the dynamics of Deffuant and Krause-Hegselmann.

I thank D. Stauffer for a critical reading of the manuscript. I gratefully acknowledge the financial support of the DFG Forschergruppe under grant FOR 339/2-1.

References

- [1] K. Sznajd-Weron and J. Sznajd, Int. J. Mod. Phys. C **11**, 1157 (2000).
- [2] W. Weidlich, *Sociodynamics; A Systematic Approach to Mathematical Modelling in the Social Sciences*. Harwood Academic Publishers, 2000.
- [3] D. Stauffer, *The Monte Carlo Method on the Physical Sciences*, edited by J. E. Gubernatis, AIP Conf. Proc. **690**, 147 (2003), cond-mat/0307133.
- [4] E. Bonabeau, G. Theraulaz and J. L. Deneubourg, Physica A **217**, 373 (1995).

- [5] R. Axelrod, J. Conflict Resolut. **41**, 203 (1997).
- [6] G. Deffuant, D. Neau, F. Amblard and G. Weisbuch, Adv. Complex Syst. **3**, 87 (2000); G. Weisbuch, G. Deffuant, F. Amblard, and J.-P. Nadal, Complexity **7**, 2002; G. Deffuant, F. Amblard, G. Weisbuch and T. Faure, Journal of Artificial Societies and Social Simulations **5**, issue 4, paper 1 (jasss.soc.surrey.ac.uk) (2002).
- [7] R. Hegselmann and U. Krause, Journal of Artificial Societies and Social Simulation **5**, issue 3, paper 2 (jasss.soc.surrey.ac.uk) (2002) and Physics A, in press (2004); U. Krause, *Soziale Dynamiken mit vielen interakteuren. Eine Problemskizze*. In U. Krause and M. Stöckler (Eds.), *Modellierung und Simulation von Dynamiken mit vielen interagierenden Akteuren*, 37-51, Bremen University, Jan. 1997.
- [8] S. Galam, J. Stat. Phys. **61**, 943 (1990) and Physica A **238**, 66 (1997).
- [9] D. Stauffer, A. O. Sousa and S. Moss de Oliveira, Int. J. Mod. Phys. C **11**, 1239 (2000).
- [10] A. T. Bernardes, D. Stauffer and J. Kertész, Eur. Phys. J. B **25**, 123 (2002), cond-mat/0111147.
- [11] I. Chang, Int. J. Mod. Phys. C **12**, 1509 (2001).
- [12] A. A. Moreira, J. S. Jr. Andrade and D. Stauffer, Int. J. Mod. Phys. C **12**, 39 (2001).
- [13] M. C. Gonzalez, A. O. Sousa and H. J. Herrmann, Int. J. Mod. Phys. C **15**, 45 (2004), cond-mat/0307537.
- [14] A. O. Sousa, cond-mat/0406390.
- [15] R. Albert and A. L. Barabási, Rev. Mod. Phys. **74**, 47 (2002); M. E. J. Newman, SIAM Review **45**, 167 (2003).
- [16] R. Ochrombel, Int. J. Mod. Phys. C **12**, 1091 (2001).
- [17] A. T. Bernardes, U. M. S. Costa, A. D. Araujo and D. Stauffer, Int. J. Mod. Phys. C **12**, 159 (2001).

- [18] D. Stauffer, A. O. Sousa and C. Schulze, Journal of Artificial Societies and Social Simulation **7**, issue 3, paper 7 (2004).
- [19] J. Bonnekoh, Int. J. Mod. Phys. C **14**, 1231 (2003).
- [20] D. Stauffer, Adv. Compl. Syst. **5**, 97 (2002) and Int. J. Mod. Phys. C **13**, 315 (2002).
- [21] A. L. Barabási and R. Albert, Science **286**, 509 (1999).
- [22] S. Fortunato, cond-mat/0406054.